SUPERFLUIDITY

Superfluidity is a consequence of having zero viscosity. Superfluids and Bose-Einstein condensates share this quality. A Bose–Einstein condensate (BEC) is a state, or phase, of a quantized medium normally obtained via confining bosons (particles that are governed by Bose-Einstein statistics and are not restricted from occupying the same state) in an external potential and cooling them to temperatures very near absolute zero. As the bosons cool, more and more of them drop into the lowest quantum state of the external potential (or "condense"). As they do, they collectively begin to exhibit macroscopic quantum properties. Through this process the fluid transforms into something that is no longer viscous, which means that it has the ability to flow without dissipating energy. At this point the fluid also loses the ability to take on homogeneous rotations. Instead, when a beaker containing a BEC (or a superfluid) is rotated, quantized vortices form throughout the fluid, but the rest of the fluid's volume remains stationary.

The BEC phase is believed to be available to any medium so long as it is made up of identical particles with integer spin (bosons), and its statistical distributions are governed by Bose-Einstein statistics. It has been observed to occur in gases, liquids, and also in solids made up of quasiparticles. In short, all fluids, whose constituents are subject to Bose-Einstein statistics, should undergo BEC condensation once the fluid's particle density and temperature are related by the following equation.

$$T_c = \left(\frac{n}{\zeta(3/2)}\right)^{2/3} \frac{2\pi\hbar^2}{mk_B} \approx 3.3125\frac{\hbar^2 n^{2/3}}{mk_B}$$

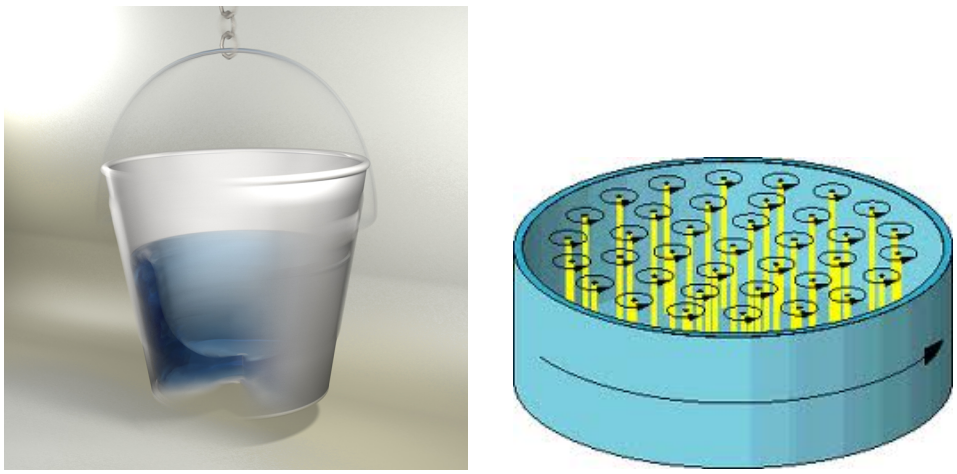
where: T_c is the critical temperature of condensation, n is the particle density, m is the mass per boson, and ζ is the Riemann zeta function.

Superfluids (all fluids exhibiting the property of superfluidity) differ from regular fluids in some important ways. Zero viscosity means that energy cannot be dissipated through the medium because there is no internal friction to do the dissipating. Metric distortions are therefore sustained. This means, among other things, that if we stir a superfluid with a spoon, creating a vortex, that vortex will not dissipate throughout the medium after we remove the spoon. In a regular fluid this kind of vortex would begin to immediately dissipate after we stopped stirring.

If we spin a bucket filled with a regular fluid the entire fluid ends us swirling about a single axis. A single vortex forms with a fluid velocity that is radially dependent. (Figure 21-1a) Near the center of the vortex the fluid moves slowly and it gradually speeds up towards the outer edge. This is not what occurs in a superfluid. Superfluids cannot take on homogeneous rotations. Instead, when we spin a bucket filled with a superfluid we produce many minute quantized vortices. (Figure 21-2b) These vortices appear throughout the medium while the rest of its volume remains stationary. The number of vortices that form is proportional to the rotational energy put into the system and $\hbar/m$, where $\hbar$ is Planck's constant and m is the mass of one quantum of the medium. The fluid

velocity is greatest at the center of each of these quantized vorticies and it dissipates radially. (Figure 21-2)

It is worth noting that if the vacuum is a superfluid, then these kinds of metric distortions may be responsible for the generation of mass particles. In general, mass generation may be built into a theory of vacuum superfluidity as a consequence of spontaneous symmetry breaking – as a consequence of the formation of quantized vortices.



(a) Regular Fluid (b) Superfluid

Figure 1-1 Rotating a regular fluid vs. a superfluid.

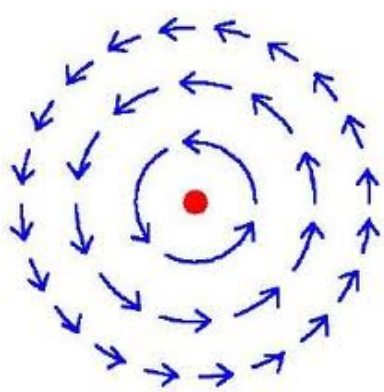


Figure 1-2 Quantized vortices have inversely radially dependent fluid velocities.

DERIVING SCHRÖDINGER'S WAVE EQUATION

Among the tools of modern physics, the Schrödinger equation has been among the most ontologically vague. It mathematically details the behavior of Ψ, but it gives us no purchase on the question of its nature. The metaphysical ambiguity beneath Ψ has given rise to dozens of interpretations of quantum mechanics, each starting from wildly different assumptions about what the wave equation tells us about the world.

Schrödinger took de Broglie's pilot wave and removed de Broglie particles from it, leaving only a wave function field defined on configuration space and significantly muddying the conceptual waters. Schrödinger tried (unsuccessfully) to make up for this lack of clarity by interpreting Ψ as a charge density.[i] Max Born subsequently interpreted it as a *probability amplitude*, whose absolute square is equal to a probability density.[ii] To the dismay of every student, this definition stuck. A *probability amplitude* is arguably just as ontologically inaccessible as the wavefunction it is supposed to elucidate. This has become the elephant in the physics classroom. We can lucidly talk about the amplitude of a probability wave, or the probability of finding an amplitude, but it is not so easy to claim that you are being lucid when talking about a *probability amplitude*.

One of the beautiful consequences of the assumption that the vacuum is a superfluid is that it dissolves the vagueness of Ψ. If the vacuum is a superfluid, then we should model it as an acoustic metric – being made up of many elastically interacting particles. This conditional carries us to the logical expectation that a wave equation will be an intrinsic descriptor of any system that is internal to the vacuum. It should be a descriptor of the state of the vacuum itself. A metric composed of particles (molecules, atoms, quanta, etc.), which collectively contribute to the emergent notions of a density field and a velocity field, necessarily has waves. These waves are analogous to sound waves, and their propagation is a function of the compressibility of the medium.

It turns out that we can indirectly and directly use the assumption that the vacuum is a superfluid (or a BEC) to derive Schrödinger's wave equation. Indirectly this can be achieved by elevating the attributes that follow from vacuum superfluidity into fundamental guiding principles. In other words, once we recognize that superfluidity suggests that energy conservation, de Broglie relations, and linearity will hold within the superfluid vacuum, we can use those constraints to derive a wave equation.

To develop this derivation note that energy conservation can be expressed through the Hamiltonian H, which is classically used to represent the total energy E of a particle in terms of the sum of its kinetic energy T, and its potential energy V.

$$E = T + V = H$$

If a particle is restricted to one-dimensional motion its energy can be described as:

$$E = \frac{p^2}{2m} + V(x,t) = H.$$

with position x, time t, mass m, and momentum p.

In three dimensions the equation becomes:

$$E = \frac{\mathbf{p} \cdot \mathbf{p}}{2m} + V(\mathbf{r}, t) = H$$

Where **r** is the position vector and **p** is the momentum vector.

When we extend this formalism to represent a large collection of particles, we have to keep track of the fact that potential energy is a function of all the relative positions of the particles (versus being a simple sum of the particle's separate potential energies). Consequently, the general equation becomes:

$$E = \sum_{n=1}^{N} \frac{\mathbf{p}_n \cdot \mathbf{p}_n}{2m_n} + V(\mathbf{r}_1, \mathbf{r}_2 \cdots \mathbf{r}_N, t) = H$$

The assumption of vacuum superfluidity also naturally captures de Broglie relations,[iii] because in any acoustic metric the energy of any phonon (a collective excitation in the medium's elastic arrangements) is proportional to the frequency v (or angular frequency $\omega = 2\pi v$) of its corresponding quantum wave packet.

$$E = hv = \hbar k$$

Furthermore, any wave can be associated with a particle such that, in one dimension, the momentum p of the particle is related to the wavelength λ by:

$$p = \frac{h}{\lambda} = \hbar k$$

and in three dimensions the relation becomes:

$$\mathbf{p} = \frac{h}{\lambda} = \hbar \mathbf{k} \ .$$

where **k** is the wave vector (the magnitude of **k** relates to the wavelength).

Finally, we can achieve generality in our derivation by noting that, in an acoustic metric, waves can be constructed via the superposition of sinusoidal plane waves. Therefore, the superposition principle is expected to apply to the waves we mean to characterize, and more generally to any acoustic metric.

With these attributes we can structure an equation that characterizes the state of the vacuum over time, without abandoning ontological clarity as to what the wave function is. We simply need to construct an equation that is capable of translating the possible Hamiltonian of a system's particles (their possible kinetic and potential energies constrained by the aforementioned characteristics of the system) into a function of the state of the system Ψ.

To trace out the steps of that derivation we would be well advised to follow Schrödinger's brilliant insight and choose to express the phase of a plane wave in the superfluid vacuum as a complex phase factor via:

$$\Psi = Ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = Ae^{i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar}$$

Next we take the first order partial derivatives of this equation with respect to space and time and we get:

$$\nabla\Psi = \frac{i}{\hbar}\mathbf{p}Ae^{i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar} = \frac{i}{\hbar}\mathbf{p}\Psi$$

$$\frac{\partial\Psi}{\partial t} = -\frac{iE}{\hbar}Ae^{i(\mathbf{p}\cdot\mathbf{r}-Et)/\hbar} = -\frac{iE}{\hbar}\Psi$$

When we combine this with our earlier expression for energy, this leads to:

$$\begin{aligned} -i\hbar\nabla\Psi = \mathbf{p}\Psi \quad &\rightarrow \quad -\frac{\hbar^2}{2m}\nabla^2\Psi = \frac{1}{2m}\mathbf{p}\cdot\mathbf{p}\Psi \\ \frac{\partial\Psi}{\partial t} = -\frac{iE}{\hbar}\Psi \quad &\rightarrow \quad i\hbar\frac{\partial\Psi}{\partial t} = E\Psi \end{aligned}$$

Now if we multiply our three-dimensional energy equation by Ψ we get:

$$E = \frac{\mathbf{p}\cdot\mathbf{p}}{2m} + V \rightarrow E\Psi = \frac{\mathbf{p}\cdot\mathbf{p}}{2m}\Psi + V\Psi$$

which naturally collapses into the following wave equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$$

This equation is exactly what Schrödinger came up with. Nevertheless, in this form it is slightly incomplete. To capture the full range of possible metric evolutions this equation must be augmented by relativistic corrections. Translating this into terms of our treatment, we can say that the current formula does not allow for inter-particle interactions between the quanta of space.

To derive the wave equation in its complete form we can take a less scenic route by directly assuming that the vacuum is a BEC whose state can be described by the wavefunction of the condensate $\psi(\vec{r})$. The particle density of this system would then be represented by $|\psi(\vec{r})|^2$. If all of the particles in this fluid have condensed to the ground state, then the total number of atoms *N* in the system will be $N = \int d\vec{r}\,|\psi(\vec{r})|^2$. Treating these particles as bosons, and using mean field theory, the energy *E* associated with the state $\psi(\vec{r})$ takes on the following expression:

$$E = \int d\vec{r}\left[\frac{\hbar^2}{2m}|\nabla\psi(\vec{r})|^2 + V(\vec{r})|\psi(\vec{r})|^2 + \frac{1}{2}U_0|\psi(\vec{r})|^4\right]$$

By minimizing this expression with respect to infinitesimal variations in $\psi(\vec{r})$, and fixing the number of particles in the system, we end up with the Gross-Pitaevskii equation (also called the nonlinear Schrödinger equation, or the time-dependent Landau-Ginzburg equation).

$$i\hbar\frac{\partial\psi(\vec{r})}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V(\vec{r}) + U_0|\psi(\vec{r})|^2\right)\psi(\vec{r})$$

where: *m* is the mass of the bosons, $V(\vec{r})$ is the external potential, *and* U_0 is representative of the inter-particle interactions.

Note that when inter-particle interactions go to zero this equation reduces to Schrödinger's original equation.

From this we see that it is possible to derive Schrödinger's wave equation from first principles. More significantly, if the assumption of vacuum superfluidity is correct, then this connection offers us the unprecedented ability to ontologically access what the wave equation means and where it comes from. The state of the vacuum is explicitly tied to this wave equation because it behaves as a superfluid. In short, the wave equation characterizes excitations in the periodicity of the medium's particles as the arrangements of those quanta elastically wave about. It describes how distortions within the metric evolve by detailing how the vacuum itself evolves.

ANALOGUE GRAVITY

The search for a quantum theory of gravity is fundamentally a search for an appropriate mathematical structure in which to simultaneously phrase both quantum questions and gravitational questions.

Matt Visser[iv]

Analogue gravity models arose out of the recognition that supersonic fluid flow can generate a *dumb hole*, which can be treated as the acoustic analogue of a *black hole*. These dumb holes are consequences of the fact that moving fluids drag sound waves along with them. If a fluid flow becomes supersonic within a certain region, the sound waves within that region will be dragged in such a way that it is impossible for them to propagate back upstream.[v] Such fluid flows form acoustic horizons (also known as sonic horizons) that are analogous to the event horizons found in general relativity. The analogy is impressively precise. It even mimics the presence of phononic Hawking radiation. Does this means that superfluidity is the key to understanding black holes? Or does superfluidity do more for us than that?

The assumption that the vacuum is an isotropic inviscid fluid (a superfluid) makes it possible to model the cosmos with equations that have ontologically accessible

interpretations. As we saw in the previous section, treating the vacuum as a superfluid allows us to specify an effective metric and accommodate a wave equation as a descriptor of that metric. In general, the phenomena that we associate with superfluidity (propagations of collective excitations, interference effects that originate from the phase of the condensate wavefunction, etc.) are encoded, in simple and compact form, by the Gross-Pitaevskii equation.[vi] We now turn our attention to the suggestion that an analogue of general relativity can be constructed from the geometrical acoustics of this non-relativistic theory.

The formal encasement of vacuum superfluidity within an acoustic metric might make it possible to capture the general relativistic notion of spacetime curvature while converting that notion into a precise mathematical and physical statement. Analogue models, which make use of geometrical acoustics, turn out to naturally "capture and accurately reflect a sufficient number of important features of general relativity."[vii] In their paper 'Analogue gravity from Bose-Einstein condensates' Carlos Barceló, S Liberati and Matt Visser mathematize the claim that superfluidity encodes a general relativistic system. I shall reproduce a summary of their treatment here.

Barceló et al. start with the Gross-Pitaevskii equation, writing it in its time dependent form:

$$i\hbar \frac{\partial}{\partial t}\psi(t,\boldsymbol{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{ext}}(\boldsymbol{x}) + \lambda\,|\psi(t,\boldsymbol{x})|^2\right)\psi(t,\boldsymbol{x})$$

and its associated effective action:

$$\mathcal{S} = \int \mathrm{d}t\,\mathrm{d}^3x \left\{\psi^*\left(i\hbar\partial_t + \frac{\hbar^2}{2m}\nabla^2 - V_{\text{ext}}(\boldsymbol{x})\right)\psi - \tfrac{1}{2}\lambda\,|\psi(t,\boldsymbol{x})|^4\right\}$$

Then they generalize the Gross-Pitaevskii equation by: (1) replacing the quartic $\frac{1}{2}\lambda\,|\psi(t, x)|^4$ by an arbirrary nonlinearty $\pi(\psi^*\psi) = \pi(|\psi|^2)$, (2) permitting the nonlinearity function to be explicitly space and time dependent: $\pi \rightarrow \pi(x, [\psi^*\psi])$ with $x \equiv (t, x)$, (3) permitting the mass to be a 3-tensor: $m \rightarrow m_{ij}$, (4) permitting that mass tensor to depend on space and time, and (5) allowing a time dependence for the confining potential $V_{ext} = V_{ext}(t, x)$.

In response to these generalizations the action now becomes:[viii]

$$\mathcal{S} = \int \mathrm{d}t\,\mathrm{d}^3x \sqrt{\det\left[{}^{(3)}h\right]}\left\{\psi^*\left(i\hbar\partial_t + \frac{\hbar^2}{2\mu}\Delta_h + \frac{\xi\hbar^2}{2\mu}{}^{(3)}R(h) - V_{\text{ext}}(t,\boldsymbol{x})\right)\psi - \pi(x,|\psi|^2)\right\}.$$

where $[{}^{(3)}h^{-1}]^{ij}$ is the inverse of the 3-metric ${}^{(3)}h_{ij}$ and $\frac{\xi\hbar^2}{2\mu}{}^{(3)}R(h)$ is the DeWitt term.

Varying this generalized action with respect to ψ^* gives us a generalized nonlinear Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(t,x) = -\frac{\hbar^2}{2\mu}\Delta_h\psi(t,x) - \frac{\xi\hbar^2}{2\mu}{}^{(3)}R(h)\psi(t,x) + V_{ext}(t,x)\psi(t,x) + \pi'(\psi^*\psi)\psi(t,x).$$

It is this generalized nonlinear Schrödinger equation that constrains the acoustic metric, as opposed to Einstein's equations of general relativity. The great insight here,

according to Barceló et al., is that this generalized nonlinear Schrödinger equation "has a (3+1)-dimensional 'effective' Lorentzian spacetime metric hiding inside it."[ix] The unexpected surprise is that the effectiveness of this Lorentzian spacetime metric appears to be momentum dependent.

Barceló et al. show that at low momenta ($k \ll m_0$) the generalized Bogoliubov dispersion relation $\omega(k) = \sqrt{m_0^2 + k^2 + \left(\frac{k^2}{2m_\infty}\right)^2}$ has the usual non-relativisitc limit: $\omega(k) = m_0 + \frac{k^2}{2m_0} + O\left(k^4\right)$.

At intermediate momenta $m_0 \ll k \ll m_\infty$ the dispersion relation takes on a relativistic form. And at large momenta $k \gg m_\infty$ the dispersion relation returns to a non-relavistic form.

$$\omega(k) = \frac{k^2}{2m_\infty} + m_\infty + O\left(k^{-2}\right).$$

The suggestion is that we should expect a regime in which phase peturbations of the wavefunction behave as though coupled to an 'effective Lorentzian metric' to be a generic consequence of vacuum superfluidity.[x] This means that it is not *a priori* implausible to mimic a gravitational field within an acoustic system.[xi] Nevertheless, to get that field to be completely accurate it may be necessary to incorporate a notion of anisotropy into the acoustics (e.g. by resolving vorticity, or curl, within the superfluid metric).

This approach has the ability to solve a problem that has plagued attempts to formulate a theory of quantum gravity. If the vacuum is a superfluid, then its quantized nature naturally cuts off the ability to resolve an 'effective metric' for gravity fields to be defined on. This both accounts for, and explains the failure to resolve such a metric. If the vacuum is quantized, then gravitational effects are emergent collective distoritions in the metric, which do not have any direct underlying coorespondence in terms of that metric. This is why "wavelengths long compared with the acoustic Compton wavelength see a Lorentzian 'effective metric,' while wavelengths short compared with the acoustic Compton wavelength probe the 'high-energy' physics (which in this situation is the non-relativistic Schrödinger equation)."[xii]

This perspective is in line with Sakharov's suggestion that gravity is no more fundamental than fluid dynamics[xiii] – a claim that our model extends to all of the *forces*. Fluid mechanics is not considered fundamental because there is a known underlying microphyics (molecular dynamics) that gives rise to it, and because it is just the low-energy low-momentum limit of that underlying microphysics.[xiv] Similiarly, our quantized model provides us with a concrete explanation of the transition from the short-distance discrete realm to the long-distance 'continuum.' In this it reveals how the spacetime manifold and metric come into focus, as a low-energy approximation (as we take an averaged over collection of space quanta). The underlying microphysics allows for

possibilities that do not conform to that approximation, but those deviations only show up at extremely high energies.

The prediction of energy dependent deviations from Lorentz invariance, which can be measured via precision tests of dispersion realtions, adds to the experimental testability of this model. At low energies the spacetime manifold should have a locally Minkowskian structure. This means that it should exhibit special relativistic effects. As the ultra high energies are approached the locally Euclidean geometry of the manifold should break down as the discreteness of spacetime begins to contribue to violations of Lorentz invariance. This will modify the dispersion relations for elementary particles.[xv] Perhaps the next task is to incorporate quantum vorticity into the efforts of the analogue gravity program, in the hopes that when we do it will lead us to a natural direct route to Einstein's equations of general relativity.

Bohmian Mechanics

"If the universe is not the way Bohm describes it, it ought to be."

John P. Briggs and F. David Peat[xvi]

"Bohmian mechanics is empirically equivalent to orthodox quantum theory."

Sheldon Goldstein

Most physicists are completely unaware of the fact that quantum mechanics is perfectly choreographed by the mathematics of Bohmian mechanics. Despite the fact that Bohm's formalism is entirely deterministic and, in my opinion, ontologically superior to any other quantum mechanical interpretation (with the possible exception of quantum space theory), it has only been fully recognized for what it is (at least at large) by the philosophers of physics. This chapter will serve as an introduction to Bohmian mechanics and its ontological implications. Among other things, this introduction will include an expansion on the historical events, or 'unfortunate accidents,' that have led to the present overwhelming ignorance of the ontological superiority of Bohmian mechanics. In part, this will serve as an explanation for why the orthodox or 'standard' interpretation of quantum mechanics is still held by the majority of physicists today – something that is arguably one of the greatest intellectual tragedies of our time. It is hoped that, at minimum, this background knowledge will free the reader from any tendence to blindly accept a interpretation of quantum mechanics simply because it is the majority view. By reclaiming a sense of healthy skepticism, and general awareness of how current views were formed, we stand to expidite future efforts to secure a theory of everything.

It is hoped that the mathematics supplied in this chapter will be considered useful to those sufficiently versed in the prerequisites. However, because my goal throughout this book has been to cover all significant concepts with as much intuitive accessibility as possible, an unfamiliarity of advance mathematics should not inhibit the reader from following the value of the discussion in this chapter. I've made it my strategy to explain the concepts behind the math independent of the actual equations. Therefore, it is hoped that the equations in this chapter will clearly satiate the hunger of those who have developed the taste for higher mathematics, while the explanations of those equations will help everyone else develop a taste for representing the world through symbols.

After we have discussed the history of Bohmian mechanics and the deterministic and ontological advantages it provides, we shall turn to a discussion of how Bohmian mechanics might be extended to envelop general relativity and therefore completely reflect the map of quantum space theory.

Bohmian mechanics[xvii] is also called the de Broglie-Bohm theory, the pilot-wave model, and the causal interpretation of quantum mechanics. Louis de Broglie originally discovered this approach in 1927. It was rediscovered by David Bohm in 1952. Technically, it could be argued that Einstein initiated the pilot-wave approach with the concept of the Führungsfeld or guiding field. (Wigner 1976, 262; Goldstein) Although he failed to complete the formalism of such an approach, Einstein independently encouraged both de Broglie and Bohm to press on with their efforts. Einstein never saw Bohm's final result but, as we are about to see, Bohm's formalism was in tune with Einstein's dream.

According to Bohm's formalization the stochastic property that manifests in the orthodox approach of quantum mechanics (in the wave collapse) is merely a reflection of the incompleteness of that approach. Because it assumes that there is more to the picture, Bohmian mechanics is often referred to as an interpretation of quantum mechanics with *hidden variables*. One way to interpret these hidden variables is to say that they represent the dimensions that we cannot see.

In addition to the Schrödinger equation, which is shared among all quantum mechanical interpretations, Bohmian mechanics is completed by the specification of actual particle positions, which evolve (in the superspatial dimensions) according to the *guiding equation*. The velocities of these particles are therefore expressed in terms of the wave function. Thus, according to Bohmian mechanics, the configuration of a system of particles evolves via a deterministic motion, which is choreographed by the wave function. The interdimensional interplay of Bohm's guiding equation and Schrödinger's wave function accounts for all the phenomena governed by nonrelativistic quantum mechanics, from spectral lines and scattering theory to superconductivity, the quantum hall effect, quantum tunneling, nonlocality, and quantum computing. (Goldstein)[xviii] Most importantly it does all of this without elevating the role of the observer to something mystical – as traditional interpretations do.

The usual measurement postulates of quantum theory naturally emerge from the combination of Schrödinger's equation with Bohm's guiding equation. This unison also elegantly restores determinism into the dynamics of physical reality. Because of this, Bohmian mechanics has recently been praised as the cure to the conceptual difficulties that have plagued quantum mechanics.

Knowing this, it may come as a surprise that mainstream physicists have failed to embrace this cure. But the fact of the matter is that most physicists haven't examined Bohmian mechanics at all. The large majority of them haven't even heard of it. This is both surprising and frustrating. If Bohmian mechanics is a cure to modern quantum mechanical philosophic complacency, then why have there been so few to study the richness of this elegant formalism? As it turns out, there are several, mostly historical, reasons that have led to this unfortunate condition. As we are about to see, those reasons turn out to be logically invalid when they are closely scrutinized. The problem is, however, that as of yet they have not been widely scrutinized.

As James Cushing puts it, Bohm's formalism has been systematically ignored and misunderstood for "reasons having more to do with politics, positivism, and sloppy thought, than for reasons central to physics." (Cushing 1994) Let's do our part to overturn this ignorance by examining the objections that have been thrown at Bohm's formalism.

The strongest historical argument against Bohm's formalism turns out to be the most embarrassing. In 1932 John von Neumann published a mathematical treatise supposedly 'proving' that Einstein's dream of discovering a deterministic complete map of Nature is impossible to achieve. This proof claimed to show that it was impossible to obtain a deterministic completion or reinterpretation of quantum theory. (von Neumann 1932) But the proof explicidly relied on locality and the assumption that the vacuum is a four-dimensional continuum. Even though locality does not hold true in our universe, a fact that was proven by John Bell,[xix] the guiding intuition of most physicists still safely clings to the old expectations of locality (and four dimensions). This is why von Neumann's proof was almost universally accepted.

An additional level of confusion has bled through the pages of history in the notion that John Bell himself argued against Bohm's hidden variables approach instead of for it. (See Wigner 1976) Bell argued that Bohm's formalism is necessarily nonlocal, but he did not argue that Bohm's formalism was an invalid representation of Nature. On the contrary, Bell was the prime supporter of Bohmian mechanics during his lifetime. Here's what Bell actually had to say on the subject.

> "But in 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the 'observer,' could be eliminated…
>
> But why then had Bohm not told me of this 'pilot wave'?...Why did von Neumann not consider it? More extraordinarily, why did people go on producing "impossibility" proofs, after 1952, and as recently as 1978?... Why is the pilot wave picture ignored in textbooks? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show us that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?" (Bell 1987, p. 160)

Bohmian mechanics clearly stands as a strict refutation of von Neumann's "proof." It is an example of the very construction that von Neumann claimed could not exist. To be fair to those that were misled by von Neumann's proof, it is true that if we confine ourselves to the rules of locality and to the seemingly justified bounds of four

dimensions, von Neumann's proof does appear to be quite valid. But when we confine ourselves in this way von Neumann's proof can no longer be said to relate to Nature. It may be a valid mathematical postulate, but it is a postulate that does not reflect the true geometry of physical reality. Clinging to such a proof is like clinging to the idea that all triangles possess a sum of 180 degrees even though you know that in regions of curved spacetime, or on extremely microscopic scales, this condition is not true.

As we have seen over and over, the tapestry of Nature is colored with the patterns of nonlocality and the brush strokes of higher dimensions. Consequently, as John Bell points out, when we consider that von Neumann intends his proof to be applicable to Nature the assumptions he uses are so unreasonable that the proof "is not merely false but *foolish*!"[xx]

Today physicists purporting to claim that a deterministic map of physical reality is 'impossible' (yes they still do this) usually cite von Neumann's proof, the Kochen-Specker theorem, which is merely an extension of von Neumann's proof, and Bell's inequality to support their claim. The fact is that Bohmian mechanics is as much a counterexample to the Kochen-Specker theorem as it is to the arguments of von Neumann. And Bell's inequality is in no way imcompatible with Bohm's construction.

An impossibility proof simply cannot be valid, no matter how compelling it appears to be, if a counterexample exists. This is what Bohmian mechanics is to von Neumann's impossibility proof and the Kochen-Specker theorem – a clear-cut formalism of exactly what they say cannot be formalized. If someone were to produce a compelling proof to support the claim that the Sun will not rise in the East and set in the West tomorrow, would you not have to conclude that their proof was flawed as it applies to the real world while you watched the Sun rise in the East and set in the West?

Furthermore, citing Bell's inequality as support of this flawed conclusion is to show a gross misunderstanding of Bell's theorem. What Bell showed was that any hidden-variables formulation of quantum mechanics must be nonlocal. He also showed that "quantum theory itself is irreducibly nonlocal."[xxi]

Therefore, it is completely nonsensical to say that any deterministic hidden-variables approach must be eliminated as a valid representation of physical reality simply because it is necessarily nonlocal. This is nonsensical because Nature *is* nonlocal. What we should be concluding is that if a theoretical map of Nature does not explicatively reveal Nature's nonlocality, it cannot be considered a complete and/or accurate map of physical reality.

Sadly people still attempt to refute Bohmian mechanics by saying that the theory cannot be correct because it is obviously and necessarily nonlocal. To the point of being annoying, this is still considered an argument against Bohmian mechanics in the public forum even though mainstream physicists have completely embraced the fact that Nature is nonlocal. The problem, it seems, is that physicists can abstractly accept that Nature is nonlocal, but they often have a difficult time letting nonlocality seep into their intuition. This conceptual lag is no doubt a consequence of dimensional tenuity – of the fact that a complete dimensional map of Nature has never before been available.

"That the guiding wave, in the general case, propagates not in ordinary three-space but in a multidimensional-configuration space is the origin of

the notorious 'nonlocality' of quantum mechanics. It is a merit of the de Broglie-Bohm version to bring this out so explicitly that it cannot be ignored."

John Bell 1987

The mathematics of Bohmian mechanics may not force a transcendence of intuition upon us, but when we comprehend the mathematics of Bohmian mechanics in conjunction with our quantized map of spacetime, nonlocality becomes readily accessible to our intuition. When we take that first intellectual step into the higher dimensional tapestries of Nature, nonlocality becomes completely unavoidable. Our assumption that the vacuum is a superfluid gives us the ability to explain the dancing wave functions in Bohm's formalism. (See Chapter 21)

The rest of the story as to why Bohmian mechanics is not currently favored as the mainstream interpretation of quantum mechanics can be traced back to orthodox philosophical intransigence. (See also chapters 13 and 17) Those that fail to comprehend, or factor in, the ontological advantages that come from the determinism of Bohmian mechanics often attempt to downplay the formalism by pointing out that it "doesn't make any predictions that differ from those of ordinary quantum mechanics." Technically, that's not much of an objection since we could equally argue that empirically "the standard theory doesn't go beyond Bohm's theory."[xxii] In fact, as physicist Hrvoje Nikolic of the Rudjer Boskovic Instutiute in Zagreb, Croatia has said, "If some historical circumstances had been only slightly different then it would have been very likely that Bohm's deterministic interpretation would have been proposed and accepted first, and would be dominating today."[xxiii]

The fact is that Bohmian mechanics completely accounts for nonrelativistic dynamics. It choreographs every dance in the quantum mechanical realm, and does so deterministically. For these reasons alone it is worthy of our attention. But it may even have the ontological power to free us from the limiting assertion of quantum mechanics.

The most controversial aspect of orthodox quantum mechanics is not the formalism itself, but rather "a further assertion to the effect that we cannot get beneath this formalism, to account for it in microscopic terms." [xxiv] The quantum formalism is touted as a "measurement" formalism. "Thus it is a phenomenological formalism describing certain macroscopic regularities."[xxv] In this it has similarities with the thermodynamic formalism. Therefore, its derivation should heavily, if not exclusively, rely on the patterns of thermodynamic formalism.

The thermodynamic formalism conceptually depends upon the assumption that the underlying behavior of a large number of microscopic constituents leads to the dynamics and interrelated properties of the larger macroscopic system. This relationship has not yet been mathematically derived from first principles. Nevertheless, the ideal gas law $PV = nRT$ relates the macroscopic properties of an ideal gas (pressure, volume, and temperature), and explains that relationship based on an underlying assumption that the system (the ideal gas) is made up of microscopic constituents (molecules) that interact elastically.[xxvi]

Several averaged-over macroscopic mathematical relations automatically follow from such assumptions. Because these mathematical relations have reliably held up in our laboratory experiments our confidence in the substrate of elastically interactive constituents (molecules) is strengthened. We now believe that we can intuitively access what lies beneath the thermodynamic formalism by accounting for its microscopic substrate. Whether or not anyone ever directly sees a molecule, or an atom, a comprehension of that microscopic realm greatly improves the intuitive access we have of physical reality.

As we derive a quantum formalism, it is in our best interest to retain the ability to "get beneath it," and explain it in microscopic terms. One way to do this is to start with the assumption that the system (the vacuum in this case) is composed of a large number of microscopic constituents that (at least to first approximation) interact elastically. When we do this a quantum formalism similar to (Bohmian mechanics) "emerges in such an inevitable manner that we are almost forced to conclude that philosophical prejudice must have played a crucial role in its nondiscovery."[xxvii]

The structure of the quantized map we have been exploring throughout this book can greatly assist our understanding of the derivation we are about to undertake. For the purposes of this derivation the word *particles* can be used to reference the quanta of spacetime. The derivation of Bohm's formalism can be done in the absence of a correlate map. Nevertheless, having a map to reference can give us an intuitive advantage during that derivation. It can help us keep track of why each step is natural and required by our considerations. Once again, let me state that the reader will not lose any continuity by simply skimming the equations that follow. The explanations are intended to be sufficient for clarity.

DERIVING BOHM'S FORMALISM

Let's begin by addressing the objective state of the wave function on the microscopic level. In this case, the term *microscopic level* references the quantum or Planck scale. If our system (a chosen domain of spacetime) is composed of N particles, then a complete description of that system will necessarily include a specification of the positions Q_i of each of those particles. On its own, the wavefunction Ψ does not provide a complete description of the state of that system. Instead, the complete description of this quantum system must be given by (Q, Ψ) where

$$Q = (Q_1, Q_2, Q_3, \ldots Q_N) \in \mathbb{R}^{3N}$$

is the configuration of the system and

$$\Psi = \Psi(q) = \Psi(q_1, q_2, q_3, \ldots, q_N),$$

a (normalized) function on the configuration space – the superspatial dimensions – is its wave function.

At this point, all we have to do in order to obtain our theory is specify the law of motion for the state (Q, Ψ). Of course, the simplest choice we can make here would be

one that is causally connected. In other words, one whose future is determined by its present specification, and more specifically whose average total state remains fixed – at least in the macroscopic sense of the familiar four dimensions of spacetime. To obtain this we simply need to choreograph the particle motions by first-order equations that assume elastic interactions. The evolution equation for Ψ is Schrödinger's equation which, as we showed in chapter 21, can be derived from the assumption of vacuum superfluidity:

$$i\hbar\frac{\partial\Psi_t}{\partial t} = H\Psi_t = -\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla^2 q_k\Psi_t + V\Psi_t$$

where Ψ is the wave function and V is the potential energy of the system.

Therefore, in keeping with our previous considerations, the evolution equation for Q should be:

$$\frac{dQ_t}{dt} = v^{\psi t} \text{ with } v^{\psi} = (v_1^{\psi}, v_2^{\psi}, v_3^{\psi}, \ldots v_N^{\psi})$$

where v^{ψ} takes the form of a (velocity) vector field on our chosen configuration space $\mathbb{R}^{3N}$. Thus the wave function Ψ reflects the motion of the particles in our system, in a macroscopic averaged-over sense, based on the underlying assumption of elastic interaction. These motions are coordinated through a vector field that is defined on our specified configuration space.

$$\Psi \longmapsto v^{\psi}$$

If we simply require time-reverse symmetry and simplicity to hold in our system (automatic necessities for a deterministic theory) then,

$$v_k^{\psi} = \frac{\hbar}{m_k}\operatorname{Im}\frac{\nabla q_k\Psi}{\Psi}$$

Notice that there are no ambiguities here. The gradient ∇ on the right—hand side is suggested by rotation invariance, the Ψ in the denominator is a consequence of homogeneity (a direct result of the fact that the wave function is to be understood projectively, which is in turn an understanding required for the Galilean invariance of Schrödinger's equation alone), the *Im* by time-reverse symmetry, which is implemented on Ψ by complex conjugation in keeping with Schrödinger's equation, and the constant in front falls directly out of the requirements for covariance under Galilean boosts.[xxviii]

Therefore, the evolution equation for Q is:

$$\frac{dQ_k}{dt} = v_k^{\psi}(Q_1, Q_2, Q_3, \ldots Q_N) \equiv \operatorname{Im}\frac{\nabla q_k\Psi}{\Psi}(Q_1, Q_2, Q_3, \ldots Q_N)$$

This completes the formalism of Bohmian mechanics that David Bohm constructed in 1952.[xxix] From one perspective, what we've done is apply the analogy of a gas being made up of elastically interacting constituents to our system of spacetime. As an

extension of de Broglie's pilot wave model[xxx] this formalism exhaustively depicts a nonrelativistic universe of *N* particles without spin.[xxxi]

Spin must be included in order to account for Fermi and Bose-Einstein statistics. The full form of the guiding equation, which is found by retaining the complex conjugate of the wave function, accounts for all the apparently paradoxical quantum phenomena associated with spin. For considerations without spin the complex conjugate of the wave function cancels because it appears in the numerator and the denominator of the equation. The full form of the evolution equation is:

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k} \operatorname{Im}\left[\frac{\Psi^* \partial_k \Psi}{\Psi^* \Psi}\right](Q_1, Q_2, Q_3, \ldots Q_N)$$

Notice that the right-hand side of the guiding equation is *J/Q*, the ratio for the quantum probability current to the quantum probability density.[xxxii] This simplicity speaks to why Bohmian mechanics has been said to be "the most naïvely obvious embedding imaginable of Schrödinger's equation into a completely coherent physical theory."[xxxiii]

Today most physicists have been brought up under the orthodox shadows of characters like Niels Bohr, Werner Heisenberg, and John von Neumann.[xxxiv] These figureheads loudly declared that a deterministic formalism of quantum mechanics is physically, philosophically, mathematically, and logically impossible.[xxxv] They set the social stage and defined the assumptions that physicists would carry for decades after them. For some reason they were so intransigently stuck to the idea that quantum theory demands radical epistemological and metaphysical innovations that they appear to have never truly considered getting beneath the quantum formalism and accounting for it in microscopic terms.[xxxvi] These men possessed extraordinary intellects, and contributed powerfully to the development of quantum mechanics, but they missed out on Bohm's obvious, elegant, and quite frankly trivial formalism.

> *"[T]he idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them… is impossible."*
>
> *Werner Heisenberg*[xxxvii]

That stubbornness is the primary reason that Bohm's *hidden variables* interpretation of quantum mechanics is not the formal interpretation taught today. This intransigence has been quite lopsided. Craig Callender notes that "For some reason or other, people often object to Bohm for reasons that they would never hold against other interpretations of QM."[xxxviii] I suspect that this has something to do with the fact that, without a map of Nature's full dimensional character, people have a tremendously difficult time elevating their intuition to a higher dimensional realm where nonlocality is automatic.

It is worth emphasizing, once again, that the biggest problem with the orthodox interpretation of quantum mechanics is that it severs the reach of our intuition. Its presumptions tautologically inhibit us from ever figuring out what is really going on.

Many physicists and philosophers have felt the poignant sting of this truncation. In fact, Schrödinger himself never quite accepted the validity or completion of the wave function based on the intuitive damage it seemed to do. He said, that the idea that the wave function "is an abstract, unintuitive mathematical construct is a scruple that almost always surfaces against new aids to thought and that carries no great message."[xxxix] If Schrödinger hadn't abandoned Bohm's particle approach he may have been led to a different opinion.

As Schrödinger voiced, a model's worth is to be measured in its ability to provide us with salient ontological clairty. Unlike traditional quantum mechanical interpretations, which restrict our intuitive reach over the realm they choreograph, Bohm's formalism (and qst) enhance our grasp of Nature. For example, despite the empirical equivalence between Bohmian mechanics and orthodox quantum theory, "there are a variety of experiments and experimental issues that don't fit comfortably within the standard quantum formalism but are easily handled by Bohmian mechanics. Among these are swell and tunneling times (Leavens, 1996), escape times and escape positions (Daumer et al., 1997a), scattering theory (Dürr et al., 2000), and quantum chaos (Cuching, 1994; Dürr et al., 1992a)."[xl]

To understand the ontological advantage that deterministic interpretations (both Bohm's interpretation and qst) have over the orthodox interpretation, let's examine the double-slit experiment (see chapters 12 and 13) from both persepctives. In the orthodox interpretation we are asked to believe that, for example, photons form an interference pattern on the back wall because they all 'magically,' in a way we cannot comprehend, manage to go through both slits. (Figure 24-1) For systems with more than two slits every photon magically manages to go through every slit.

In order to accept this interpretation we have to do more than abandon our notion of a particle - we have to accept that this 'magic' is truly just that – magic. This is because we have to accept that it really is impossible for us to ever have intuitive access to the process that causes photons, electrons, etc., to form interference patterns in the double-slit experiment – that it is impossible for us to comprehend, understand, or ever know what's really going on during these experiments.

Figure 1-3 Do single photons/electrons really go through both slits?

The prevailing orthodox interpretation pushes this worldview upon us. Richard Feynman explains this by saying that the interference pattern made during the double-slit experiment is "a phenomenon which is impossible, *absolutely* impossible, to explain in

any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery."[xli] Feynman later said, "Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it."[xlii]

If this were true it would be a pretty large pill to swallow. Thankfully, it is not true. As we have seen, "Bohmian mechanics is just such a deeper explanation."[xliii] Bohmian mechanics (and in identical fashion the assumption of vacuum superfluidity) provides a deeper explanation of the double-slit experiment.[xliv] Once we discover this, the parrallel between those that hold the orthodox interpretation of quantum mechanics and the robed puppet masters of orthodox religions becomes apparent. Both preach that we are incapable of getting to know or discovering the truth for ourselves - that we should just give up and embrace unquestioning faith.

I hope that it is obvious by now that such an attitude is detrimental to the personal scientific quest. More importantly, I hope that it is easy to see that when it comes to its effects on humanity as a whole, the abandonment of this quest would be catastrophic. Bohm's interpretation frees us from the outdated sins of orthodox unquestioning faith. It shows us that the path of the photon in our double-slit experiment reflects an interference pattern because the motion of that photon is governed by the wave function. Parts of the wave function pass through both slits while the particle passes through one slit. The parts of the wave function that pass through separate slits interfere with each other, developing an interference profile that guides the particle on its way.

The interference pattern we see is, therefore, an unavoidable consequence of nonlocality – of the fact that spacetime is quantized. It is not a magical, unexplainable effect. If the particles are emitted one by one, then this interference pattern still builds up over time – provided that the trajectories of the ensemble have a random (in four dimensions) distribution, which we can denote as $\rho = \langle \Psi \rangle^2$.

If every particle were to follow completely identical trajectories, then they would all end up at one spot, creating a single bright spot on our photographic plate (or the wall). In Nature this is not a real possibility for photons because the substrate of spacetime is composed of interactive quanta. Before two particles can follow identical trajectories, identical trajectories must exist. This is not to be expected because the metric of a superfluid vacuum is not static. The quanta are constantly mixing about. This is why the best information about available four dimensional trajectories is given by $\rho = \langle \Psi \rangle^2$. More generally, this explains why extremely precise information about a trajectory in the familiar four dimensions can at best be described statistically or probabilistically.

For two photons to follow the exact same path through space (identical trajectories) the positions and velocities of all the intermittent space quanta along that path (the hidden variables) would also have to be identical. On macroscopic scales this is extremely improbable. So from the ontological vantage of Bohmian mechanics (and qst) the interference pattern we see in the double-slit experiment is exactly what we should expect. That's a rather significant improvement over the orthodox claim - that we should just accept the double-slit experiment as something that will just never make sense.

This beautiful formalism, which we have now intuitively accessed from the conceptual vantage of our new quantized map, completely accounts for four dimensional randomness, absolute uncertainty, familiar (macroscopic) reality, and the presence of the wave function within the vacuum, while explaining away the mysteries of wave-particle duality, wavefunction collapse, and providing a precise, sharp foundation for the uncertainty principle. It was derived as the simple and natural consequence of the assumption that the vacuum is a superfluid.

Technically, the assumption that the constituents of that vacuum interact with perfect elasticity was fixed in our derivation – as was the assumption that the system does not interact with any other/exterior environments. This assumption is quite reasonable (except during the Big Bang) if we choose our system to be the whole universe. This elasticity is reflected in the first-order evolution equation and it is also encoded in the Schrödinger equation. In other words, the transformation $\rho^{\psi} \rightarrow \rho^{\psi^t}$ arises directly from Schrödinger's equation. If these evolutions are indeed compactable, then

$$(\rho^{\psi})_t = \rho^{\psi^t}$$

is equivariant. Therefore, under the time evolution ρ^{ψ} retains its form as a function of Ψ.

The conclusion that $\rho\Psi = |\Psi|^2$ is equivariant follows immediately from the observation that the quantum probability current $J^{\psi} = |\Psi|^2 v^{\psi}$, so that the continuity equation

$$\frac{\partial \rho}{\partial t} + div\left(\rho v^{\Psi}\right) = 0$$

is satisfied by the density $\rho_t = |\Psi|^2$. As a consequence, we find that if $\rho\ (q, t_0) = |\Psi(q, t_0)|^2$ at some time t_0, then $\rho\ (q, t) = |\Psi\ (q, t)|^2$ for all t.[xlv]

All of this is based upon the first-order expectation of perfect elasticity. To be sure, this assumption is quite valid as a first-order approximation of the system (since it completely maps the quantum mechanical realm.) But we are not strictly bound to this assumption. If we loosen this assumption, to allow for second-order inelasticity, we may be able to make room for relativistic effects.

GOING BEYOND BOHM'S FORMALISM

Now that we can see beneath the formalism it might be possible for us to discover how to extend it into a relativitic theory. If we start with Bohm's formalism of quantum mechanics, remembering that it resulted from the assumption of an underlying substrate composed of microscopic parts that interact elastically, at least to the first order, and then re-derive the formalism based on the additional assumption that a tiny second-order degree of four-dimensional inelasticity also needs to be taken into account (with a magnitude that will, of course, be determined by the Planck constants (and therefore relate to the gravitational constant G), quantum mechanics and general relativity might become two different resolutions of the same map. Such a formalism would be the

mathematical backbone of the quantized map that we have been exploring in this book.[xlvi] It has become a central aim of the superfluid vacuum community.

The crucial conceptual key here comes from the two ways in which we justify this small measure of geometric inelasticity. One of those ways comes from the cascading levels of dimensional hierarchy. As we imagine an eleven dimensional version of our map we see our universe and the quanta that compose its macroscopic substrate. When the quanta that build the substrate of our universe collide, their internal subquanta are geometrically reorganized due to those collisions. This geometric adjustment transfers some of the energy (which, as we have seen, is defined as a geometric entity) of the quanta to the internal subquanta. Since the structural difference between quanta and their internal subquanta are separated by many orders of magnitude, the geometric transfer from one level to the next is proportionally miniscule, but it is not zero.

The second, and perhaps more significant, justification comes from inter-particle interactions. Schrödinger's equation, the one used in Bohm's derivation, assumed zero inter-particle interactions.

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$$

An improvement might be made by reworking through Bohm's calculations starting with the Gross-Pitavaskii equation:

$$i\hbar\frac{\partial\psi(\vec{r})}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V(\vec{r}) + U_0|\psi(\vec{r})|^2\right)\psi(\vec{r})$$

where: *m* is the mass of the bosons, $V(\vec{r})$ is the external potential, *and* U_0 is representative of the inter-particle interactions.

Over large spans of time and across macroscopic reaches of space this extra term may contribute low-amplitude metric distortions that are responsible for spacetime density gradients (although I suspect the complete solution will also require the incorporation of superfluid vorticity).[xlvii] Such overlooked factors might turn out to weave together the fabrics of the two great traditions of physics into one complete, seamless formalism. Getting beneath quantum mechanics and accounting for it in microscopic terms has the potential to bring a revolution comparable to the one we gained when we described the world as being composed of atoms. If successful, it may carry us to a new philosophical age.

"If Bohm's physics, or one similar to it, should become the main thrust of physics in the future, the dances of the East and West could blend in exquisite harmony."

Gary Zukav[xlviii]

[i] Moore, W.J. (1992). *Schrödinger: Life and Thought*. Cambridge University Press. p. 219. ISBN 0-521-43767-9.

[ii] Ibid., p. 220.

[iii] L. de Broglie, *Recherches sur la théorie des quanta* (Researches on the quantum theory), Thesis (Paris), 1924; L. de Broglie, *Ann. Phys.* (Paris) **3**, 22 (1925).

[iv] Carlos Barceló, Stefano Liberati and Matt Visser, "Analogue Gravity", Living Rev. Relativity, **8**, (2005), 12. [Online Article]: cited [<11-26-2012>], p. 77, http://www.livingreviews.org/lrr-2005-12

[v] Carlos Barceló, Stefano Liberati and Matt Visser, "Analogue Gravity", Living Rev. Relativity, **8**, (2005), 12. [Online Article]: cited [<11-26-2012>], p. 7, http://www.livingreviews.org/lrr-2005-12

[vi] Carlos Barceló, S Liberati and Matt Visser, "Analogue gravity from Bose-Einstein condensates" Institute of Physics Publishing, Class. Quantum Grav. **18** (2001), PII: S0264-9381(01)18993-8, p. 1140.

[vii] Carlos Barceló, Stefano Liberati and Matt Visser, "Analogue Gravity", Living Rev. Relativity, **8**, (2005), 12. [Online Article]: cited [<11-26-2012>], p. 6, http://www.livingreviews.org/lrr-2005-12

[viii] Here Δ_h is the three-dimensional Laplacian defined by $\Delta_h \psi = \frac{1}{\sqrt{\det\left[{}^{(3)}h\right]}} \nabla_i \left(\sqrt{\det\left[{}^{(3)}h\right]} \left[{}^{(3)}h^{-1}\right]^{ij} \nabla_j \psi \right)$; Carlos Barceló, S Liberati and Matt Visser, "Analogue gravity from Bose-Einstein condensates" Institute of Physics Publishing, Class. Quantum Grav. **18** (2001), PII: S0264-9381(01)18993-8, p. 1142.

[ix] Carlos Barceló, S Liberati and Matt Visser, "Analogue gravity from Bose-Einstein condensates" Institute of Physics Publishing, Class. Quantum Grav. **18** (2001), PII: S0264-9381(01)18993-8, p. 1142.

[x] Ibid., p. 1153.

[xi] Ibid.

[xii] Ibid.

[xiii] Sakharov, A.D., "Vacuum quantum fluctuations in curved space and the theory of gravitation:, *Sov. Phys. Dokl*., **12**. 1040-1041, (1968); Visser, M., "Sakharov's induced gravity: A modern perspective," *Mod. Phys. Lett. A*, **17**, 977-992, (2002). Related online version (cited on 31 May 2005): http://arXiv.org/abs/gr-qc/0204062.

[xiv] Carlos Barceló, Stefano Liberati and Matt Visser, "Analogue Gravity", Living Rev. Relativity, **8**, (2005), 12. [Online Article]: cited [<11-26-2012>], p. 76, http://www.livingreviews.org/lrr-2005-12

[xv] Ibid., p. 76-77.

[xvi] John P. Briggs and F. David Peat, 'Looking Glass Universe.'

[xvii] Much of this chapter follows Sheldon Goldstein's publication 'Bohmian Mechanics' found in the online Stanford Encyclopedia Of Philosophy.

[xviii] For more information on the identical success of Bohmian mechanics with the traditional quantum formalism see: Detlef Dürr, Sheldon Goldstein and Nino Zanghi, 'Quantum Physics

Without Quantum Philosophy,' Physical Review Letters, vol. 93, p 090402; Ward Struyve and Hans Westerman, Proceedings of the Royal Society A, vol. 463, p. 3115; D. Bohm, 'Proof that probability density approaches $<\Psi>^2$ in causal interpretation of quantum theory,' Physical Review 89, (1953), 458-466; D. Bohm, 'A suggested interpretation of the quantum theory in terms of "hidden" variables,' Physical Rev. 85 (1952), p 166-193; M. Daumer, D. Dürr, S. Goldstein, and N. Zanghi, 'A survey of Bohmian mechanics, Il Nuovo Cimento.'

[xix] The most conclusive proof widely referenced today is found in Aspect, A., Dalibard, J., and Roger, G., 1982, "Experimental test of Bell's inequalities using time-varying analyzers,' Phys. Rev. Lett. 49: 1804-1807.

[xx] Mermin 1993, p. 805; Sheldon Goldstein.

[xxi] Sheldon Goldstein.

[xxii] Mark Buchanan, 'No dice,' New Scientist, March 22, 2008 pp. 28-31.

[xxiii] Ibid.

[xxiv] Detlef Dürr, Sheldon Goldstein, and Nino Zanghí, 'Quantum Physics Without Quantum Philosophy,' p. 4. The mathematics to follow, as well as much of the remaining discussion, heavily relies on this abstract.

[xxv] Ibid., p. 4.

[xxvi] Boyle's law is another example of this. Technically, pressure and temperature are macroscopic properties that also rely on this underlying assumption. These properties result from the group behavior of a large number of elastically interactive molecules in motion.

[xxvii] Detlef Dürr, Sheldon Goldstein, and Nino Zanghí, 'Quantum Physics Without Quantum Philosophy,' p. 4.

[xxviii] Ibid., pp. 5-6.

[xxix] David Bohm, 1952.

[xxx] L. de Broglie, 1927.

[xxxi] Of course in the limit $\hbar/m \rightarrow 0$, the Bohm motion Q_t approaches the classical motion. See: D. Bohm and B. Hiley, 'The Undivided Universe: an Ontological Interpretation of Quantum Theory,' Routledge & Kegan Paul, London, 1993; Detlef Dürr, Sheldon Goldstein, and Nino Zanghí, 'Quantum Physics Without Quantum Philosophy,' p. 7.

[xxxii] Sheldon Goldstein, 'Bohmian Mechanics.' For further examples of how easily spin can be dealt with in the Bohmian formalism see: J. S. Bell, 1966, 447-452; D. Bohm, 1952, 166-193; D. Dürr et all 'A survey of Bohmian mechanics, Il Nuovo Vimento' and 'Bohmian mechanics, identical particles, parastatistics, and anyons', In preparation.

[xxxiii] D. Dürr, et al.

[xxxiv] See J. von Neumann 1932, R. T. Beyer, 1955 pp. 324-325; J. S. Bell 1982, 989-999; J.S. Bell 1987, pp. 159-168.)

[xxxv] J. von Neumann 1932; J. S. Bell 1982, 1987.

[xxxvi] D. Dürr et al., pp. 4-6.

[xxxvii] 1958, p. 129; ibid.

[xxxviii] Craig Callender, Review, Brit. J. Phil. Sci. 49 (1998), 332-337.

[xxxix] Schrödinger, E., 1935, 23: 807-812, 923-828, 844-849.

[xl] Sheldon Goldstein 'Bohmian Mechanics.'

[xli] Feynman, R. P., Leighton, R. B., and Sands, M., 1963, 'The Feynman Lectures on Physics, I, New York: Addison-Wesley; Sheldon Goldstein, 'Bohmian Mechanics.'

[xlii] Feynman, R. P., 1967, 'The Character of Physical Law,' Cambridge MA: MIT Press; Sheldon Goldstein 'Bohmian Mechanics.

[xliii] Sheldon Goldstein, 'Bohmian Mechanics.'

[xliv] The whole menagerie of quantum mechanical effects gain deeper explanations in both Bohm's formalism and even more so in our completely intuitive quantized map.

[xlv] D. Dürr et al., 'Quantum Physics Without Quantum Philosophy,' p. 8.

[xlvi] For more on how the formalism of Bohmian mechanics naturally points to a formalism richer than the standard orthodox theory see: Anthony Valentini of the Perimeter Institute in Waterloo, Ontario, Journal of Physics A: Mathematical and theoretical, vol. 40, p. 3285.

[xlvii] The reworked formalism of Bohmian mechanics, in which second-order geometric inelasticity is included in the underlying assumptions, is being attempted.

[xlviii] Gary Zukav, 'The Dancing Wu Li Masters,' p. 310.

**The first three sections of this pdf (Superfluidity, Deriving Schrödinger's Wave Equation and Analogue Gravity) were excerpted from Chapter 21 in Einstein's Intuition. The final sections (Deriving Bohm's Formalism, Going Beyond Bohm's Formalism) were excerpted from Chapter 24.

A full pdf preprint of Einstein's Intuition is available upon request. For more on Analogue Gravity see:

http://einsteinsintuition.com/wp-content/uploads/Analogue-gravity-from-BECs.pdf